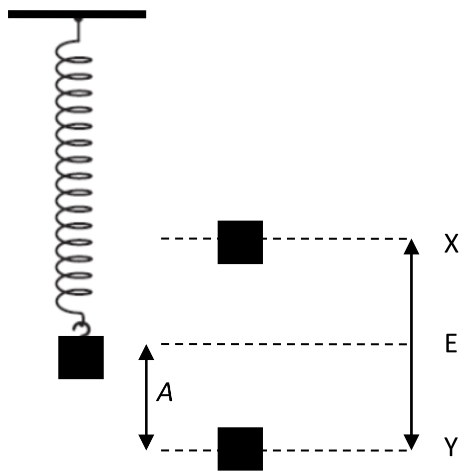
Teacher notes
Topic A

The power of the work-kinetic energy relation

A mass m is in equilibrium attached to the end of a vertical spring of spring constant k. When displaced downwards by a distance A, the mass oscillates in between extreme positions X and Y (where the speed is zero).



What is the work done by the tension in the spring as the mass moves from X to Y?

Suppose we have forgotten the work-kinetic energy relation. How do we do this problem? Then we will remember it and see its great power!

At E the spring is extended by a distance e given by

$$mg = ke \Rightarrow e = \frac{mg}{k}$$

Assume for simplicity that $A < e$. Then, at X the spring extension is $e - A$ and at Y it is $e + A$.

The work from X to Y is the **negative** of the change in elastic potential energy and equals

$$-\left(\frac{1}{2}k(e+A)^2-\frac{1}{2}k(e-A)^2\right)=-2kAe$$

The reason for the negative sign in front of the big bracket is that the change in elastic potential energy is the work done by an external agent moving the mass. The work done by tension is the opposite. We should have expected that the work done would be negative because from X to Y the tension is directed upwards but the displacement is downwards and $\cos 180^\circ = -1$.

The total work of the tension force from X to Y is then:

$$W_{\mathrm{T}} = -2kAe$$

Since $e=\frac{mg}{k}$ the work done by the tension from X to Y is

$$\begin{aligned} W_{\mathrm{T}} &= -2kAe \\ &= -2kA\frac{mg}{k} \\ &= -2mgA \end{aligned}$$

This is the opposite of the work done by the weight from X to Y. This suggests there must be an easier way to solve this problem!

Indeed, there is and makes use of the **work-kinetic energy relation**. We know that

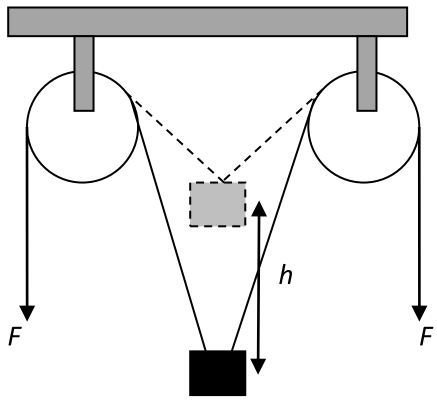
$$W_{\mathrm{net}} = \Delta E_{\mathrm{K}}$$

From X to Y, $\Delta E_{\mathrm{K}} = 0$ hence $W_{\mathrm{net}} = 0$. But $W_{\mathrm{net}} = W_{\mathrm{T}} + W_{mg}$ hence $W_{\mathrm{T}} = -W_{mg}$. Clearly, $W_{mg} = mg \times 2A$ and so $W_{\mathrm{T}} = -2mgA$.

It is clear that the use of the work-kinetic energy relation is much preferable than the detailed approach. It is simple, direct and avoids complicated issues such as the minus signs of the detailed approach.

The same idea can be applied to this problem:

A mass *m* is raised at constant speed a vertical distance *h* by the action of two equal forces *F* applied to each of the vertical strings.



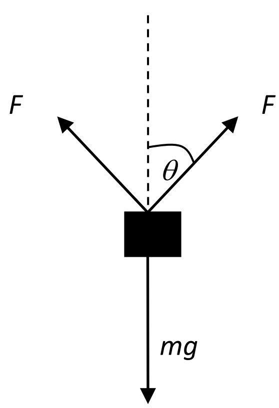
What is the work done by *F*?

$W_{net} = \Delta E_K$

$\Delta E_K = 0$ because the speed is constant, hence $W_{net} = 0$. But $W_{net} = 2W_F + W_{mg}$ hence $W_F = -\frac{1}{2}W_{mg}$.

Clearly, $W_{mg} = -mgh$ and so $W_F = \frac{1}{2}mgh$.

Using the work-kinetic energy relation is **much preferable** to the detailed solution below:



$$2F\cos\theta = mg \Rightarrow F = \frac{mg}{2\cos\theta}$$

Thus, the force *F* is not constant since the angle keeps changing as the mass is raised. However, the work done is

$$W_F = Fh\cos\theta = \frac{mg}{2\cos\theta} \times h \times \cos\theta = \frac{mgh}{2}$$

The power of the work-kinetic energy relation is enormous!